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INSTYTUT AUTOMATYZACJI PROCESÓW TECHNOLOGICZNYCH
I ZINTEGROWANYCH SYSTEMÓW WYTWARZANIA

Andrzej BUCHACZ, Marek PŁACZEK*
Institute of Engineering Processes Automation and Integrated Manufacturing Systems,
Faculty of Mechanical Engineering, Silesian University of Technology, Gliwice, Poland
* marek.placzek@polsl.pl

A SERIES OF MATHEMATICAL MODELS OF MECHATRONIC SYSTEM WITH PIEZOELECTRIC ACTUATOR

Abstract: Paper presents a process of modelling and investigation of a mechatronic system with piezoelectric transducer used as a vibration actuator – reverse piezoelectric effect is applied in considered system. A series of mathematical models of this system is presented. Characteristic that describes relation between amplitude of the system's vibration and parameters of harmonic voltage that supplied piezoelectric actuator is assigned on the basis of corrected approximate Galerkin method. Obtained results are juxtaposed and the most appropriate mathematical model of this system is chosen.

1. Introduction

Materials with piezoelectric properties called smart materials are widely used as sensors or actuators. It is possible because both the direct and reverse piezoelectric effect can be used. Piezoelectric transducer generates electric voltage when it is deformed or deforms when an electric voltage is applied [6]. In the first case it can be used as a sensor, while in the second case it is an actuator. Nowadays, there are a lot of commercial applications of direct and reverse piezoelectric effects [5,7,8].

It is very important to use precise mathematical model of system with piezoelectric transducer used as a sensor or an actuator in order to obtain required system's operation and dynamic characteristic. Therefore, a process of modelling and development of a mathematical model of system with piezoelectric transducer used as a passive vibration damper was presented in papers [1-3]. A series of mathematical models of this system was presented and approximate Galerkin method was used to analyze it. The approximate method was verify and corrected, so obtained result could be treated as very precise [4]. Now, a series of discrete – continuous mathematical models of system with PZT transducer used as an actuator is presented. Results obtained using each of mathematical models are juxtaposed to select the most appropriate model of this system. A possibility to determine the impact of properties of all system's components on its dynamic characteristic and minimal complexity of the mathematical model are established criteria.

2. Considered system and assumptions

The considered system with piezoelectric actuator is presented in Fig. 1. It is a cantilever beam with a PZT transducer glued on the beam's surface. The transducer is supplied by an external voltage source, so it works as an actuator of the beam's flexural vibration.

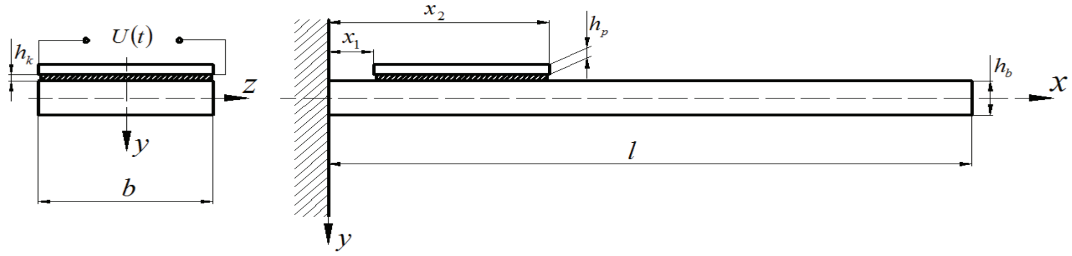

Fig.1. System with piezoelectric actuator

Dynamic characteristic α_V describes relation between deflection $y(x,t)$ of the beam's free end $(x = l)$ and electric voltage that supply the actuator:

$$y(x,t) = \alpha_V \cdot U(t), \tag{1}$$

where:

$$U(t) = U_0 \cdot \cos\omega t\,. \tag{2}$$

In agreement with assumptions of the approximate Galerkin method the equation of the beam's deflection was assumed as [4]:

$$y(x,t) = A \cdot \sin\frac{(2n-1)\pi x}{2l} \cdot \cos\omega t\,,\quad n = 1,2,3... \tag{3}$$

A is amplitude of vibration.

Internal resistance and electric capacitance of the piezoelectric transducer were included so the actuator supplied by the external harmonic voltage was modelled as a linear RC series electric circuit and described by equation:

$$R_P C_P \frac{\partial U_C(t)}{\partial t} + U_C(t) = U(t), \tag{4}$$

where: R_P and C_P are electric resistance and capacitance of the transducer. $U_C(t)$ is an electric voltage on the capacitor.

Rheological properties of the beam and glue layer between the transducer and beam's surface were introduced using Kelvin – Voigt model of materials.

3. A series of mathematical models

A series of discrete – continuous mathematical models of the considered system was created using equation of the piezoelectric actuator (5) and equation of the beam's motion designated in accordance with d'Alembert's principle, taking into account arrangement of forces and bending moments acting on a part of the system. The cantilever beam was modelled as a Bernoulli-Euler beam.

In the first mathematical model the glue layer between the beam's surface and actuator was neglected. An equality of the beam and actuator's strains was assumed. Arrangement of forces and bending moments acting in this system is presented in Fig. 2.

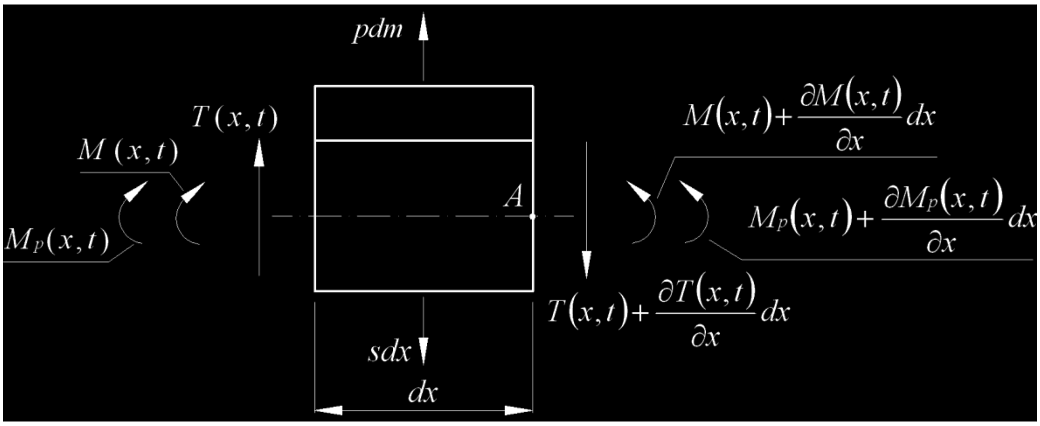

Fig.2. Arrangement of forces and bending moments acting in the system for the first model

$T(x,t)$ and $M(x,t)$ are transverse forces and bending moments that replace action of the cut-off part of the beam. $M_P(x,t)$ is the bending moment generated by the actuator as a result of applied voltage. Obtained equation of the beam's motion is:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\frac{E_b^{*} J_b}{\rho_b b h_b}\left(1+\eta_b \frac{\partial}{\partial t}\right)\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{(h_b + h_p) c_{11}^{E} b h_p}{2\rho_b b h_b} \cdot \frac{\partial^2}{\partial x^2}\left[H S_1(x,t) - H\lambda_1(t)\right], \quad (5)$$

where: $S_1(x,t)$ is the beam's surface strain, $\lambda_1(t)$ is a strain of the free actuator that occurs as a result of externally applied voltage. J_b, ρ_b and η_b are moment of inertia, density and structural damping coefficient and E_b^{*} is a substitute Young modulus of the beam [1]. c_{11}^{E} is Young modulus of the actuator measured under constant electric field. A Heaviside function:

$$H = H(x - x_1) - H(x - x_2), \quad (6)$$

was introduced to curb a working space of the actuator to partition from x_1 to x_2.

In the second model influence of the glue layer between the beam and actuator was included. A pure shear of this layer was assumed. Arrangement of forces and bending moments acting in this system is presented in Fig. 3. Obtained equation of the beam's motion:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\frac{E_b^{\,*} J_b}{\rho_b b h_b}\left(1+\eta_b \frac{\partial}{\partial t}\right)\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{G^* l_p}{2\rho_b h_k}\left(1+\eta_k \frac{\partial}{\partial t}\right)\cdot \frac{\partial}{\partial x}\left[H\left(\varepsilon_k(x,t)-\lambda_1(t)-\varepsilon_b(x,t)\right)\right], \quad (7)$$

where: G^*, η_k are a substitute shear modulus and structural damping coefficient of the glue layer. ε_k and ε_b are the glue layer and beam's strains.

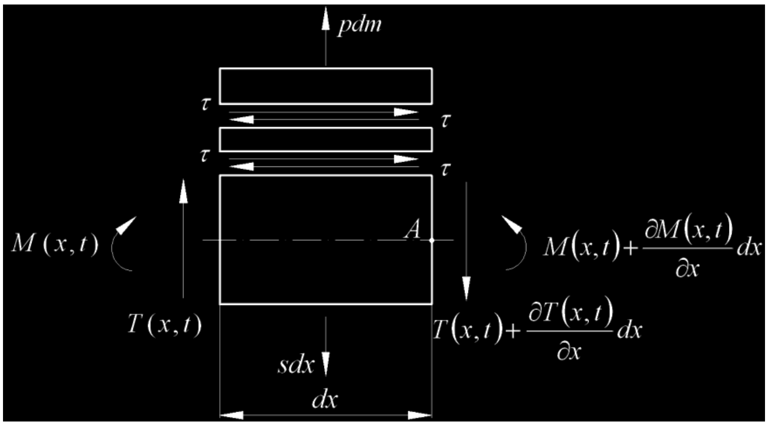

Fig.3. Arrangement of forces and bending moments acting in the system for the second model

In the third mathematical model the considered system was modelled as a combined beam and process of eccentric tension of the glue layer was considered [2]. A substitute cross-section of the composite beam was introduced and stress of the system's elements was designated. Obtained equation of the beam's motion is:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\frac{E_b^{\,*} J_b}{\rho_b b h_b}\left(1+\eta_b \frac{\partial}{\partial t}\right)\frac{\partial^4 y(x,t)}{\partial x^4} + \frac{T_7}{2\rho_b}\left(1+\eta_k \frac{\partial}{\partial t}\right)\cdot \frac{\partial^2}{\partial x^2}\left[HT_5\varepsilon_b(x,t)-HT_6\lambda_1(t)\right], \quad (8)$$

where:

$$T_5 = c_{11}^{\ E} b h_p T_1\left(y_w - 0{,}5h_p\right) - E_b J_w z_b^{-1} - h_b T_1 c_{11}^{\ E} b h_p J_w z_b^{-1}\left[A_w^{-1} + J_w^{-1} z_b\left(0{,}5h_p - y_w\right)\right], \quad (9)$$

$$T_6 = c_{11}^{\ E} b h_p \left(T_2+1\right)\left(y_w - 0{,}5h_p\right) - \left(T_2+1\right) c_{11}^{\ E} b h_p J_w z_b^{-1}\left[A_w^{-1} + J_w^{-1} z_b\left(0{,}5h_p - y_w\right)\right], \quad (10)$$

$$T_7 = \frac{E_k b^{-1}\left[E_k^{-1} c_{11}^{\ E} h_p\left(y_w - 0{,}5h_p\right) + h_k\left(y_w - h_p - 0{,}5h_k\right)\right]}{E_b h_b\left[\frac{h_b^{\,2}}{12} + \left(h_p + h_k + 0{,}5h_b - y_w\right)^2\right] + E_k h_k\left[\frac{h_k^{\,2}}{12} + \left(y_w - h_p - 0{,}5h_k\right)^2\right] + c_{11}^{\ E} h_p\left[\frac{h_p^{\,2}}{12} + \left(y_w - 0{,}5h_p\right)^2\right]}, \quad (11)$$

$$z_k = -y_w + h_p, \qquad z_b = -y_w + h_p + h_k, \quad (12)$$

$$T_1 = \frac{z_k}{z_b\left[1-\left(A_w E_b\right)^{-1} c_{11}^{\ E} b h_p\left(z_k z_b^{-1} - 1\right)\right]}, \quad T_2 = \frac{c_{11}^{\ E} b h_p A_w^{-1}\left(z_k z_b^{-1} - 1\right)}{E_b\left[1-\left(E_b A_w\right)^{-1} c_{11}^{\ E} b h_p\left(z_k z_b^{-1} - 1\right)\right]}. \quad (13)$$

In the last mathematical model the considered system was also modelled as a combined beam but the impact of the actuator to the beam was described by the bending moment generated as a result of applied electric voltage. Obtained equation of the beam's motion is:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = -\frac{E_b^* J_b}{\rho_b b h_b}\left(1+\eta_b \frac{\partial}{\partial t}\right)\frac{\partial^4 y(x,t)}{\partial x^4} + \left(\frac{h_p + h_b}{2} + h_k\right)\frac{c_{11}^E h_p}{\rho_b h_b}\frac{\partial^2}{\partial x^2}\left[T_1 \varepsilon_b(x,t) H - (T_2+1)\lambda_1(t) H\right]. \tag{15}$$

4. Obtained results

Using designated mathematical models modulus of the dynamic characteristic Y_V was calculated and presented in Fig. 4 for the first three natural frequencies of the mechanical subsystem $(n = 1,2,3)$. Parameters of the system are presented in Tab. 1.

Tab. 1. Geometrical and material parameters of the mechanical subsystem, actuator and glue layer

$l = 0{,}24[m]$	$b = 0{,}04[m]$	$h_b = 0{,}002[m]$	$h_k = 0{,}0001[m]$
$x_1 = 0{,}01[m]$	$h_p = 0{,}001[m]$	$x_2 = 0{,}09[m]$	$E_b = 210000[MPa]$
$\rho_b = 7850\left[\frac{kg}{m^3}\right]$	$\eta_b = 8\cdot 10^{-5}[s]$	$G = 1000\cdot 10^6[Pa]$	$\eta_k = 10^{-3}[s]$
$d_{31} = -240\cdot 10^{-12}\left[\frac{m}{V}\right]$	$e_{33}^T = 2900\cdot\varepsilon_0\left[\frac{F}{m}\right]$	$s_{11}^E = \frac{1}{c_{11}^E} = 17\cdot 10^{-12}\left[\frac{m^2}{N}\right]$	$\rho_p = 7450\left[\frac{kg}{m^3}\right]$

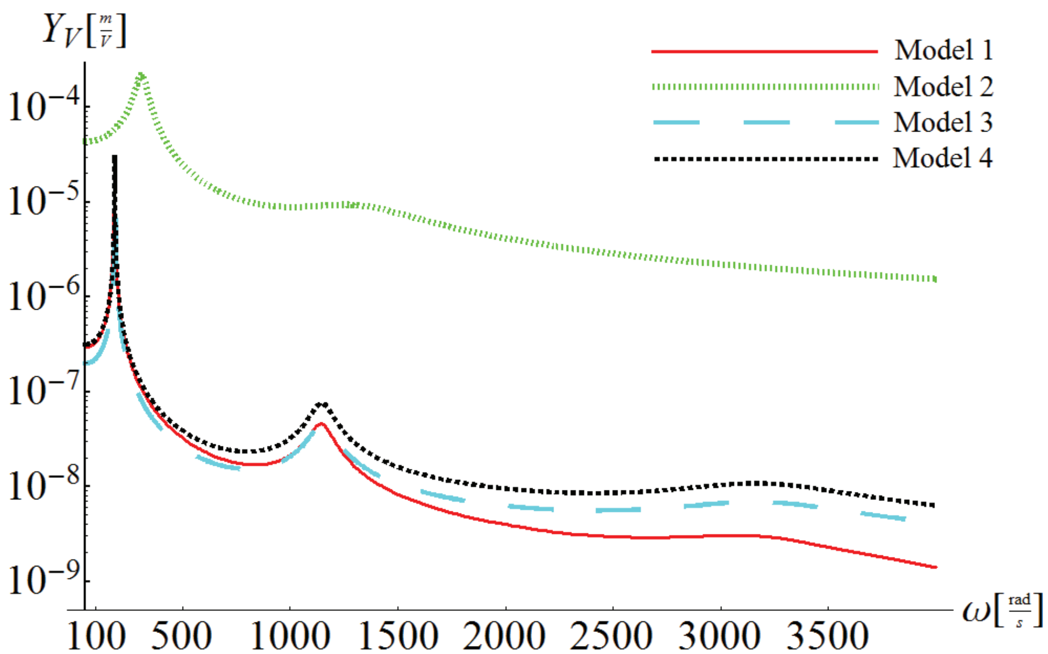

Fig.4. Dynamic characteristic of the considered mechatronic system, for n=1,2,3

The dynamic characteristic is presented in a half logarithmic scale in order to present obtained results precisely.

5. Conclusions

It was proved that after correction the approximate Galerkin method can be used to analyze mechatronic systems with piezoelectric actuators. Taking into account verification of the approximate method presented in the previous publication [4], obtained results can be treated as very precise. Mathematical model with bending moment and eccentric tension of the glue layer (model 4) can be treated as the most optimal model – properties of all system's components are taken into account (including influence of the glue layer), while this model is quite simple. Assumption about pure shear of the glue layer (model 2) leads to the idealization of the effectiveness of the system.

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