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Iwona PAPROCKA\*, Angelika DZIEGIEL

<sup>1</sup>Silesian University of Technology, Institute of Engineering Processes Automation and  
Integrated Manufacturing Systems, Gliwice

\*iwona.paprocka@polsl.pl

## THE STATISTICAL CONTROL OF THE MEASURING PROCESS CAPABILITY OF VERTICAL DISPLACEMENT OF THE HEAD RESTRAINT - THE FIRST PART, THEORY

**Abstract:** In this paper, various techniques of the statistical process control (SPT) are presented namely: histogram, control cards, machine capability and process capability. The histogram is used to make sure that the process is statistically stable. The control cards are used to determine whether a current deviation from the norm is caused by a random or specific action. The control cards enable to indicate the process which is deregulated. In the paper, the guidelines of the application of the process capability indicator to determine whether the process is able to meet the quality requirements imposed by a customer are presented. The guidelines of the application of the machine capability indicator to indicate whether a task allocated to a machine is executed with a sufficient accuracy at the specified time are presented.

### 1. Introduction

It happens that the statistical process control (SPC) is considered as a form of control, which is intended merely to separate the good quality products from defective products. However, the SPC states as the system of preventing irregularities. Systems that monitor quality parameters of production, are unreliable and expensive. Therefore less costly methods of gathering information on the quantities and types of possible defects are searched for. Statistical analysis of a process allows for control, monitoring and evaluation of the process quality basing on the measurement of a certain number of items, which are called a sample. Performing measurements for the selected sample reduces the quality control costs.

It is important that defective products are immediately withdrawn from production, and the resulting defects are eliminated. For this purpose, the knowledge about the modes (sources) of formation of deviations is needed. The SPC allows one to objectively "look at" production, provides the knowledge about the efficiency of the process, as well as any deviations from the norm and modes of their formation [11]. The SPC allows one to associate the effects and modes such as: the method of obtaining measurement data, model specifications, economic framework, random, unknown technical modes of disruptions [5, 8].

In this paper, various techniques of the SPC are presented. In the second paper, under the title of: “The statistical control of the measuring process capability of vertical displacement of the head restraint - the second part, case study” factors that influence over the control process of vertical displacement of the head restraint are identified. In the second paper, the techniques of the SPC are applied to identify disturbances that occur in the process.

## 2. Six Sigma

The Six Sigma is a method which predicts the appearance of defects before their occurrence. The Six Sigma method is based on the acquiring a high number of process data for the subsequent use them to achieve the maximum quality. Sigma is mathematically defined by the formula:

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N}} \quad (1)$$

$x_i$  – the measurement result of the characteristics of product  $i$ ,  $\bar{x}$  – the mean value of the  $N$  measurements,  $N$  – the size of the population.

In practice, however, the standard deviation  $s$  of a random selected sample from the population is computed:

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} \quad (2)$$

$x_i$  – the measurement result of the characteristics of product  $i$  of the sample,  $\bar{x}$  – the average value of  $n$  measurements,  $n$  – the sample size.

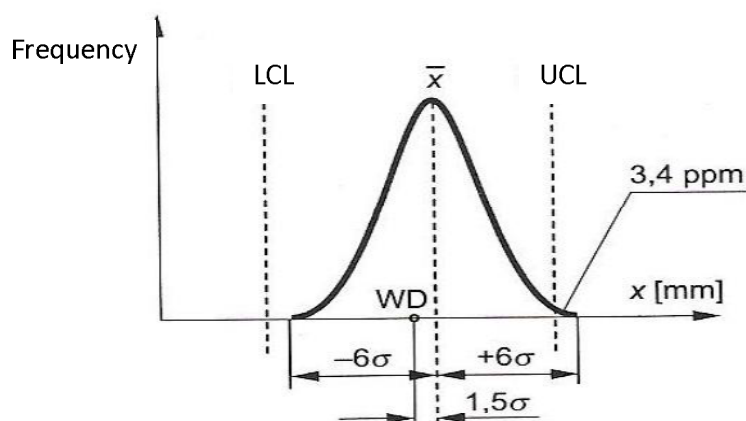


Fig. 1. Graphical representation of the Six Sigma rule [1] (UCL - an upper control line, LCL - a lower control line).

The tolerance should equal  $\pm 6 \sigma$  in the method of Six Sigma (Fig.1) [6,7]. When the process is centered ( $\bar{x}$  is in the middle of the interval  $\langle LCL, UCL \rangle$ ) only 0.0000002% of products are beyond the limits of tolerance, this means that only 2 products are defective per million of possibilities. Obtaining such the result is impossible since the process is influenced by several factors. The factors may be random or systematic, so that the process is drifting, i.e. the curve is displaced. According to the Six Sigma, the shift equals  $\pm 1.5 \sigma$ . For such the shift 0.00034%, of products are beyond the specification limits, which is 3.4 defects per million of possibilities. To obtain this result, the concept of Six Sigma was proposed [1]. The concept of Six Sigma is applied in the SPC.

### 3. Statistical Process Control

Executing the SPC, the information about when and where corrective and perfecting actions should be implemented is given. This is due to the fact that each process has a variability in its nature. The variability arises from a variety of factors that often have influence over a process. The SPC allows for monitoring whether the process is statistically controllable (predictable) also for distinguishing disturbances that occur in the process. Using the SPC, the number of products inconsistent with the requirements can be reduced. Quality control may be performed:

- offline - activities in the area of prevention, without ongoing monitoring the process.
- online - activities in the area of operational control of the process. It is used after exhausting control offline tools.

The range of quality control can concern:

- small circuits of quality control, including the production area. They may include the control of a workstation, or several workstations related to each other. They analyze technological operations performed on the workstations,
- local circuits of quality control, including few stages of a product life cycle,
- extensive circuits of quality control, including the entire product life cycle.

The following techniques of the SPC are described in the next Subsections:

- a histogram,
- a control card,
- a process capability -  $C_p, C_{pk}$ ,
- a machine capability -  $C_m, C_{mk}$ .

#### 3.1. Histogram

A histogram is the graphical representation of the empirical distribution of features. The values of tested features in a sample can appear a number of times which corresponds to the size of the classes compartments.

Before using control cards and establishing control limits (UCL and LCL), one should make sure that the process is statistically stable (Fig. 2). When fluctuations and instability are presented in the chart (Fig. 3), one must first remove the special modes and only then begin to develop a control card.

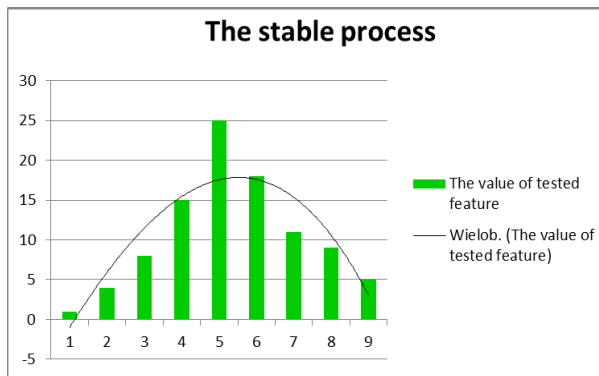


Fig. 2. The correct distribution, the process is statistically stable.

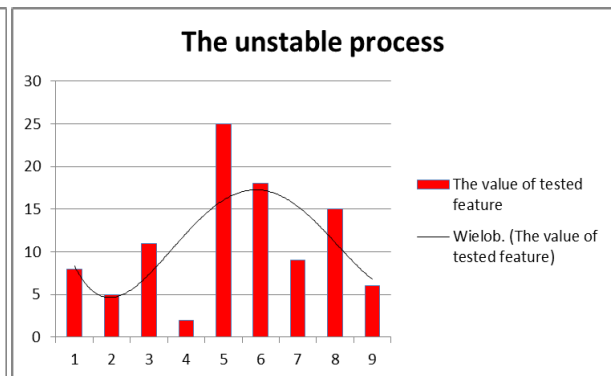


Fig. 3. The process statistically unstable.

### 3.2. Control Cards

Shewhart's control card is a method of monitoring and regulation based on graphical diagrams. The diagram enables to observe the course of a process in a systematic manner. Collected regularly data from samples are put in the diagram (Fig. 4).

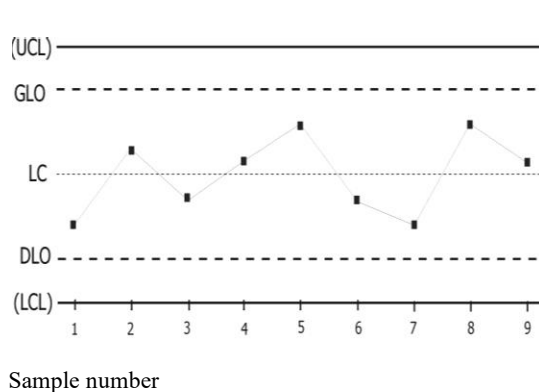


Fig. 4. Not exceeded control limits in Shewhart's control card [2]  
where: LC - a central line, GLO - an upper warning line, DLO - a lower warning line.

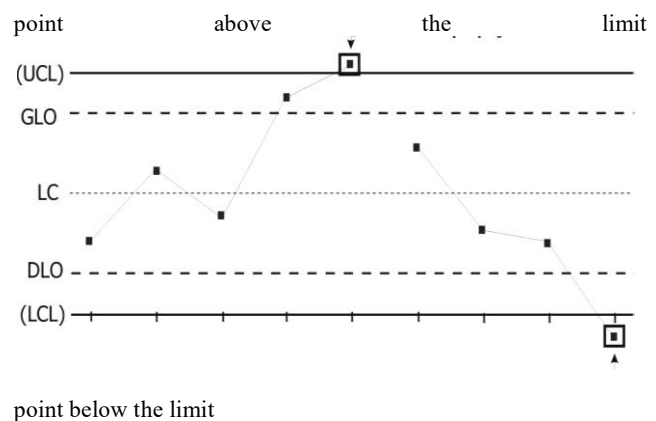


Fig. 5. Diagram presenting the danger of the process deregulation [2]. (control limits are exceeded)

Ordinal numbers of samples are given in the horizontal coordinate. The control limits are plotted on the chart. Basing on the control cards one can determine whether the current deviation from the norm is caused by a random or specific action. It is urgent to react when the control card identifies a special deviation from the norm. This means that the process may be deregulated and the defect must be eliminated. Figure 4 does not contain violations, in other words, the sample data satisfy standards. The following modes of instability in processes can be distinguished:

- normal: those that are "incorporated in the process," they affect the instability of the process but the process is still (statistically) predictable (Fig. 4). The instability is caused by normal modes.

- special: those that interfere the natural stability of the process and cause it's dysregulation. In the control cards, instability is represented by exciding the limits or by trends (Fig 5). When the problem is solved the process returns to the normal state.

Symptoms indicating a threat to the process are:

- the occurrence of points outside the control lines (Fig. 5).
- the presence of specific sequences of successive points.

The following control cards for measurable features are distinguished [2]:  $\bar{X}$  card or  $\bar{X}$  card,  $X_{\max} - X_{\min}$  ( $R$ ) card,  $\bar{X} - R$  card and  $\bar{X} - S$  card

The Following control cards for immeasurable features are distinguished [2]: the card of fraction of non-conforming units, the card of numbers of non-compliant units, the card of number of inconsistencies and the card of number of inconsistencies per unit.

### 3.3. Process capability

The process capability states as the degree of fulfilling the quality requirements imposed by a customer to a supplier. It is important that the supplier is able to meet the certain minimum process capability for selected features. Because of the susceptibility of the process to produce products outside the accepted tolerance, the following indicators are defined:  $C_p$  i  $C_{pk}$ . Other process capability indicators are presented in [9, 10].

$C_p$  indicator determines whether the process is able to meet the demand of the tolerance interval width, also known as the precision of the process.

$$C_p = \frac{\overbrace{USL - LSL}^B}{\underbrace{6\sigma}_A} \quad (3)$$

where:  $\sigma$  – standard deviation.

Correctly, the average value should coincide with the center of the tolerance interval, while the standard deviation equal to a maximum of one-sixth of the field (Fig. 6). When the tested sample is described by a normal distribution, it is essential that at least  $6\sigma$ , or 99.74% of all products must belong to the tolerance interval. The objective is that 8, 10 or even 12 values of  $\sigma$  belong to the tolerance interval. The width of the process is reduced in relation to the width of the tolerance interval, it means that the process has a greater capability [4].

When the width of the process equals to the tolerance interval ( $C_p = 1$ ), the defectiveness equals 0.27% according to the characteristics of normal distribution. Meanwhile the global standard is to achieve  $C_p = 1.33$ , for which the defectiveness equals 0.0063%.

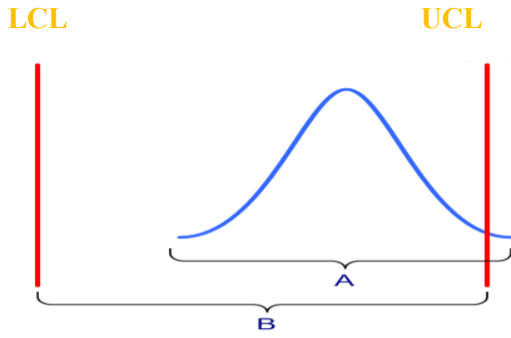


Fig. 6. The determination of the process capability [3].

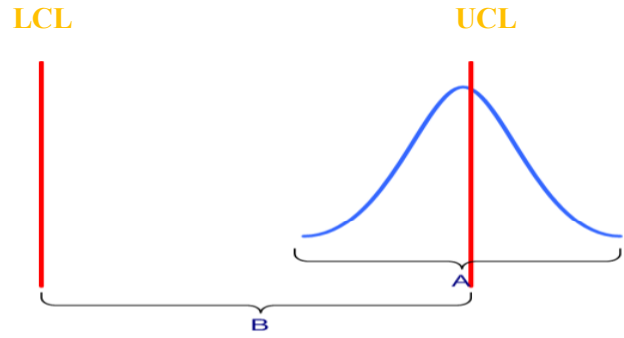


Fig. 7. Changing the position (centering) of the process does not affect the value of  $C_p$  [3].

The process width can be less than the tolerance interval (eg.  $C_p = 2$ ), and 90% of defective products can appear at the same time when the process is moved outside the tolerance interval (Fig. 7). Therefore, it is necessary to determine another indicator which is responsible for the centering of the process. This indicator takes into account the average value of the process and examines the capability of the both halves of the chart [4].

$C_{pk}$  indicator represents the average value of the process (average of the average). Graphically, the average value of the process is presented as a dotted line (Fig. 8). Assuming that each two hours a sample consisting of five pieces, is taken in the control process. The average value is calculated for each subsequent subgroup (sample) and thus a set of averages is achieved. The average of average values corresponds to the center of the process and is called as the position measure.

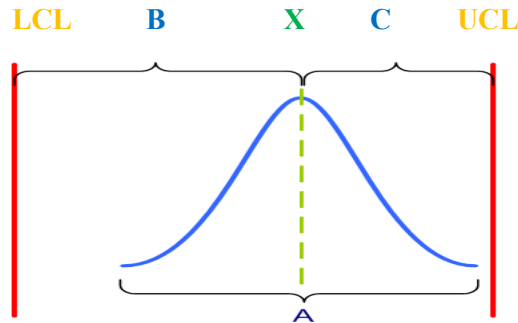


Fig. 8.  $C_{pk}$  indicator determination (based on[3]) ( $X$ – the mean value of the sample).

$$C_{pk} = \min \left\{ \underbrace{\frac{USL - X}{3\sigma}}_{A/2}; \underbrace{\frac{X - LSL}{3\sigma}}_{A/2} \right\} \tag{4}$$

The  $C_p$  and  $C_{pk}$  indicators are interrelated. The  $C_p$  states as the potential capability of the process since it represents what can be achieved when the process is perfectly centered. In

contrast, the  $C_{pk}$  states as the real ability since it informs about the actual level of defects that may occur due to the wide dispersion and improper centering of the process.

*Tab. 1. The  $C_p$  and  $C_{pk}$  indicators values interpretation.*

$C_p$	$C_{pk}$	Interpretation
2	2	The ideal case. The process is very well centered and highly capable.
2	0,5	The centering of the process is necessary. The process is characterised by a high potential ( $C_p$ ), but is poorly centered (a low value of the $C_{pk}$ ).
0,5	0,5	The dispersion of the process is large, it is necessary to reduce it. The process is well centered ( $C_p = C_{pk}$ ).
1,8	-0,5	There is the large potential of the process, but due to the poor centering, most of the results will be out of the specification (the average is out of the one of the specification limits).

The low value of the  $C_p$  informs about the large dispersion. The low value of the  $C_{pk}$  and the high value of the  $C_p$  inform about a weak centering (Tab. 1). Equation 4 indicates in which direction the process is wrong shifted. If the first part has a lower value than the other - the process is shifted to the right. If the reverse process occurs then the process is shifted to the left. After interpretation of the  $C_p$  and  $C_{pk}$  indicators values the necessary adjustment of the process is made.

### 3.4. Machine capability

The machine capability states as the number of possible to produce items in a given period of time. Evaluation of the results indicates whether a task allocated to a machine is executed with a sufficient accuracy at the specified time. In order to check whether the machine is working well following tests are performed:

- the test of machine capability checks the machine efficiency, without taking into account external factors.
- the test of process capability checks the machine efficiency taking into account external factors.

Tests are performed in order to:

- check whether a result is described by a normal distribution,
- control the position and dispersion of the result,
- ensure whether the position and dispersion complies with the requirements.

The machine capability is described by indicators:  $C_m$ ,  $C_{mk}$ , MC [3].

$C_m = C_p$  (3) states as the comparison between the dispersion and the width of the tolerance interval.  $C_m > 1$  is required.

$C_{mk}$  states as the adjusted capacity of the machine which represents where the result is in relation to the limits (5).  $C_{mk}$  value indicates if the condition of tolerance is satisfied.  $C_{mk} \geq 1$  is required.

MC evaluates the centering of the target value (6), i.e. how far the target value is from the average value. MC indicator determines how many percent of the average value is far from the target value.  $MC = \pm 10\%$  is the required value of the indicator.

$$C_{mk} = \frac{X - LSL}{6\sigma \cdot 2} = \frac{USL - LSL}{6\sigma \cdot 2} \quad (5)$$

$$MC = \frac{X - WD}{USL - LSL} \cdot 100 \quad (6)$$

where: WD – the center of the tolerance (the specification limits) [3] (Fig.1)

#### 4. Conclusions

Thanks to continuous monitoring and ongoing verification of the results, one can also control works executed on the assembly line, check whether robots and machines are not deregulated, and whether employees made no mistakes. Such the control may also be used to improve and modernize the production process. The Control of the process enables to estimate the degree of meeting the requirements by the process.

Thanks to the quality audits, problems that still exist, are identified. The problems can be eliminated using qualitative tools. The appropriate usage of control cards enables to achieve the higher quality of the process with lower unit costs and higher process capability at the same time. In this paper, various techniques of the SPC were presented. In the second paper, under the title of: “The statistical control of the measuring process capability of vertical displacement of the head restraint - the second part, case study” factors that influence over the control process of vertical displacement of the head restraint are identified. In the second paper, the techniques of the SPC are applied to examine whether the process is statistically controllable and to identify disturbances that occur in the process.

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