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AN EXAMPLE OF USE OF AN APPROXIMATE METHOD
TO ANALYSE MECHANICAL SYSTEMS

Abstract: Paper presents an example of an approximate method application to the analysis of mechanical systems. The analysed system is a simply supported vibrating beam, excited by an external harmonic force. The approximate Galerkin method is used to analyse it and to designate a dynamic flexibility of the system. The possibility of using the Galerkin method and its verification was a first step to processes of analysis and synthesis of simple and complex mechatronic systems with piezoelectric transducers connected with mechanical subsystems. In this paper the simply supported beam is analysed with the external force applied in a distance η from the one of the beam's end. It causes the analysis more complicated and exactness of the approximate method must be verified in this case.

1. Introduction

Application of the approximate Galerkin method to the analysis of vibrating mechanical and mechatronic systems was presented in previous author's publications [2]. It was used to designate the dynamic flexibility of considered systems. Its exactness was verified on the basis of a cantilever beam analysis and it was proved that it can be used to the analysis of mechatronic systems. The optimal (taking into account maximal precision and minimal complication of required calculations) mathematical model of considered mechatronic systems was identified. Mechatronic systems with piezoelectric transducers used as vibration dampers or actuators were analysed [2]. Obtained results are an introduction to the synthesis task of simply and complex mechatronic systems. The aim of the future works will be to develop a mathematical algorithm useful to design such kind of systems with desired values of natural frequencies and required dynamic characteristics. Presented work is a part of this research that had to be done in order to verify the possibility of precise analysis of presented way of mechanical system fixing using the approximate method.

2. The exact and approximate methods application

In order to analyse the considered system presented in Fig. 1, the exact Fourier's method of separation of variables was used first. Then the approximate method was used and obtained results were juxtaposed. It was assumed that the considered beam is made of steel, its length is 1,083 [*m*] and it has a square cross section with sides equal to 0,02 [*m*].
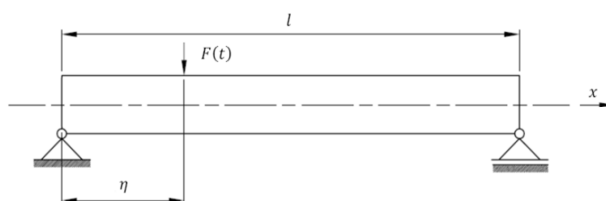

Fig.1. The considered mechanical system

Using well known method of separation of variables the dynamic flexibility of the system can be calculate. It is important to point out, that boundary conditions of the considered system describe the beam's displacements $y(x,t)$:

$$y^{(1)}(0,t) = 0, \qquad y^{(2)}(l,t) = 0, \tag{1}$$

and bending moments:

$$\frac{\partial^2 y^{(1)}}{\partial x^2}(0,t) = 0, \qquad \frac{\partial^2 y^{(2)}}{\partial x^2}(l,t) = 0. \tag{2}$$

where superscripts (1) and (2) denote left and right part of the beam separated by the point of the external force application. A condition of deflections, angles and bending moments as well as the balance of forces must be met at this point:

$$y^{(1)}(\eta,t) = y^{(2)}(\eta,t), \tag{3}$$

$$\frac{\partial y^{(1)}}{\partial x}(\eta,t) = \frac{\partial y^{(2)}}{\partial x}(\eta,t), \tag{4}$$

$$\frac{\partial^2 y^{(1)}}{\partial x^2}(\eta,t) = \frac{\partial^2 y^{(2)}}{\partial x^2}(\eta,t), \tag{5}$$

$$\frac{\partial^3 y^{(1)}}{\partial x^3}(\eta,t) = \frac{\partial^3 y^{(2)}}{\partial x^3}(\eta,t) = \frac{F(t)}{EI}, \tag{6}$$

where *E* denotes Young's modulus and *I* moment of inertia of the beam's cross section.
Taking into account those conditions an eigenvalue problem is reduced to solve the following boundary problem:

$$\begin{cases} \dfrac{\partial^4 X(x)}{\partial x^4} - k_n{}^4 X(x) = 0 \\[2mm] X^{(1)}(0) = 0 \\[2mm] X^{(2)}(l) = 0 \\[2mm] \dfrac{\partial^2 X^{(1)}(0)}{\partial x^2} = 0 \\[2mm] \dfrac{\partial^2 X^{(2)}(l)}{\partial x^2} = 0 \\[2mm] X^{(1)}(\eta) = X^{(2)}(\eta) \\[2mm] \dfrac{\partial X^{(1)}(\eta)}{\partial x} = \dfrac{\partial X^{(2)}(\eta)}{\partial x} \\[2mm] \dfrac{\partial^2 X^{(1)}(\eta)}{\partial x^2} = \dfrac{\partial^2 X^{(2)}(\eta)}{\partial x^2} \\[2mm] \dfrac{\partial^3 X^{(1)}(\eta)}{\partial x^3} = \dfrac{\partial^3 X^{(2)}(\eta)}{\partial x^3} = \dfrac{F(t)}{EI} \end{cases} \tag{7}$$

where:

$$k_n = \frac{n\pi}{l}, \qquad n = 1,2,3\ldots \tag{8}$$

$X(x)$ denotes a function of the beam's deflection. After calculations equations of the system's dynamic flexibility, separately for both sides of the beam, can be described as:

$$\alpha^{(1)} = \frac{1}{2k^3 EI}\left[\frac{-\sin kx \cdot \sin k(\eta - l)}{\sin k\eta \cdot (\cot k\eta \cdot \sin k(\eta - l) - \cos k(\eta - l))} + \frac{\sinh kx \cdot \sinh k(\eta - l)}{\sinh k\eta \cdot (\coth k\eta \cdot \sinh k(\eta - l) - \cosh k(\eta - l))}\right],$$
$$\tag{9}$$

$$\alpha^{(2)} = \frac{1}{2k^3 EI}\left[\frac{-\sin k(x - l)}{\cot k\eta \cdot \sin k(\eta - l) - \cos k(\eta - l)} + \frac{\sinh k(x - l)}{(\coth k\eta \cdot \sinh k(\eta - l) - \cosh k(\eta - l))}\right], \tag{10}$$

The absolute value of obtained dynamic flexibility of the system is presented in Fig.2. It was calculated with an assumption that the external force is applied at the distance 0,3[m] from the left end of the beam.

Assumptions of the approximate Galerkin method were presented in details in previous author's publications [2]. In this work the assumed solution of the beam's differential equation of motion was assumed as:

$$y(x, t) = A\sum_{n=1}^{\infty} \sin \frac{n\pi x}{l} \cos \omega t, \tag{11}$$

Assumed equation (13) was introduced into differential equation of the beam's motion and, after transformations, equation of the system's dynamic flexibility was obtained:

$$\alpha = \frac{1}{l\left(EIk^4 - \rho A\omega^2\right)}.$$
(12)

Obtained results are presented in Fig. 2, juxtaposed with results of the exact method.
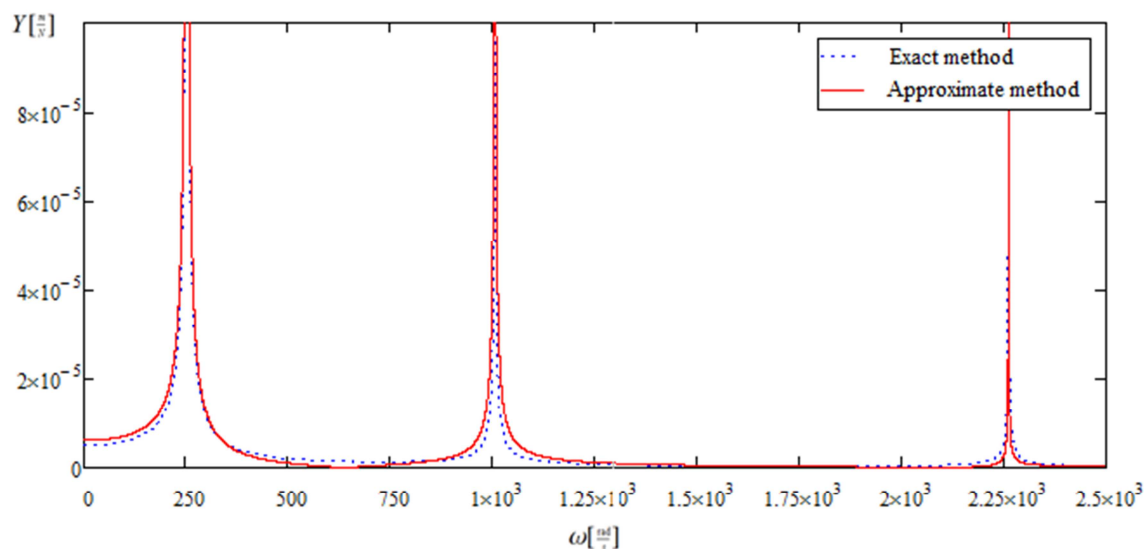

Fig.2. The system's dynamic flexibility – exact and approximate methods

As it is presented in Fig. 2 obtained results are very similar for both methods.

3. Conclusion

This work is concerned with a long-time work of Gliwice centre about analysis and synthesis of vibrating mechanical systems, as well as determination of the characteristics [l,3]. The proposed research problem is thus a continuation and extension of previous works whose goal is to develop new mathematical models and universal methods of analysis and synthesis of simple and complex mechatronic systems. Presented mechanical system was treated as one of the possible subsystems of mechatronic systems.

References

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