SELECTED ENGINEERING PROBLEMS
NUMBER 4

INSTITUTE OF ENGINEERING PROCESSES AUTOMATION
AND INTEGRATED MANUFACTURING SYSTEMS

Andrzej BUCHACZ[1], Yury STEPANOV[2]*, Gennady BARSUKOV[2],

Alexander MIKHEYEV[2]

[1] Institute of Engineering Processes Automation and Integrated Manufacturing Systems,
Faculty of Mechanical Engineering, Silesian University of Technology, Gliwice, Poland
[2] State University – Educational Scientific Production Complex, Orel, Russia
*yury057@yandex.ru

# DEFINITION OF ABRASIVE WATER JET CUTTING CAPACITY TAKING INTO ACCOUNT ABRASIVE GRAIN PROPERTIES

**Abstract:** The authors of the paper obtained dependences for the definition of abrasive water jet cutting capacity taking into account abrasive grain properties permitting choosing optimal cutting conditions.

## 1. Introduction

The micro-photo analysis of surfaces machined after abrasive water jet cutting shows that the destruction mechanism of materials with different physic-mechanical properties is about the same [1]. Microcutting is predominant. Thus, for example, when cutting aluminum which is small inclined to a brittle failure, the specific removal is relatively high. Microcutting is carried out at the single application of force of a flying abrasive particle sufficient in size for the detachment of a particle from material work-in-process and having a higher hardness in comparison with it (Fig. 1).

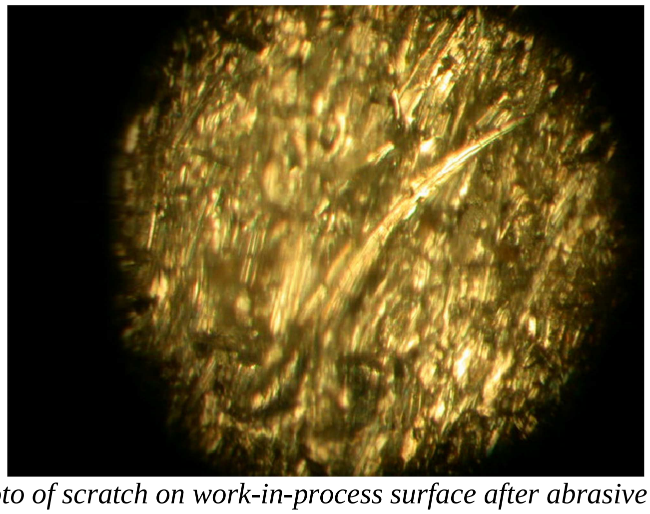


*Fig. 1. Microphoto of scratch on work-in-process surface after abrasive water jet cutting*

The capacity of chips taken away by a single abrasive grain defines a abrasive water jet cutting performance. In this connection it is necessary to know a path of a particle penetration into work-in-process pieces determining depth *h*, width *b* and length *L* of the scratch caused by lugs of a grain microrelief . In the paper [2] it was determined that the average value of angles 2β at a vertex decreases with the decrease of a grain number. The value of angles 2β is within the limits 40-150°. And at the same time the percentage of acute angles makes up 12-25%.

## 2. Modeling of penetration of an abrasive grain into material

Let us examine the problem of penetration of an abrasive grain into material when the angle between a contact surface and symmetry axis of penetrated particle б is small. At the determination of a path of a single grain motion we shall take into account the rotation of an abrasive particle round a centre of mass.

To solve the problem let us make the following assumptions:

1) value Δ of grain penetration into material is less of its radius;

2) the angular velocity of grain rotation round a symmetry axis is absent, but round a centre of mass is equal to zero at the initial moment of a contact;

3) at the initial moment of a contact with a material surface the velocity vector of a grain concurs with the axis of its symmetry;

4) let us approximate the form of an abrasive grain by two cones having a common base the vertexes of which are equally spaced on different sides.

Now let us determine the overpressure acting on the surface of a contact of an abrasive grain with material by the dependence [3]:

$$p = v^2 \frac{\rho_0}{b(\nu-2)}\left[\frac{\nu-2}{\nu}\left(a^{\frac{\nu}{2}}-1\right)+b(\nu-2)a^{\frac{\nu}{2}}-\left(a^{\frac{\nu}{2}}-1\right)\right]\sin^2\beta+ \\ +\left(a^{\frac{\nu}{2}}-1\right)\left(p_0+\frac{\tau_0}{\nu(1+\mu)}\right), \qquad (1)$$

where *a,b* - factors $a = \dfrac{1}{1-b}$ ; $b = \dfrac{\rho_0}{\rho(t)} = const$

c0 - initial density of work material;

c(t) - current density of work material in the area of a contact with an abrasive grain;

$V_0$ – axial component of the velocity of a grain penetration into material;

$\tau_0$ – yield point of work material;

μ – Poisson’s ratio for work material;

ν – factor, $\nu = \dfrac{2\mu}{1+\mu}$;

$p_0$ – initial pressure on the surface of a grain contact with material;

β – vertex angle of an abrasive grain.

Let us determine the projection area of abrasive grain $S_y$ and $S_z$ on plane $y\xi$ and $z\xi$ of moving coordinates $yz\xi$ (Fig. 2).

Let us put down the equation of the intersection of an abrasive grain tapered surface with a barrier surface:

$$\xi^2 + \left[\left(\frac{d}{2} - \Delta\right) - z\sin\alpha\right]^2 = tg^2\beta\left(\frac{d}{2tg\beta} - z\right)^2, \qquad (2)$$

where $\xi$, $z$ - current coordinates of the abrasive grain at an arbitrary point of time;

*d* – grain diameter;

$\Delta$ - depth of grain penetration into a surface.

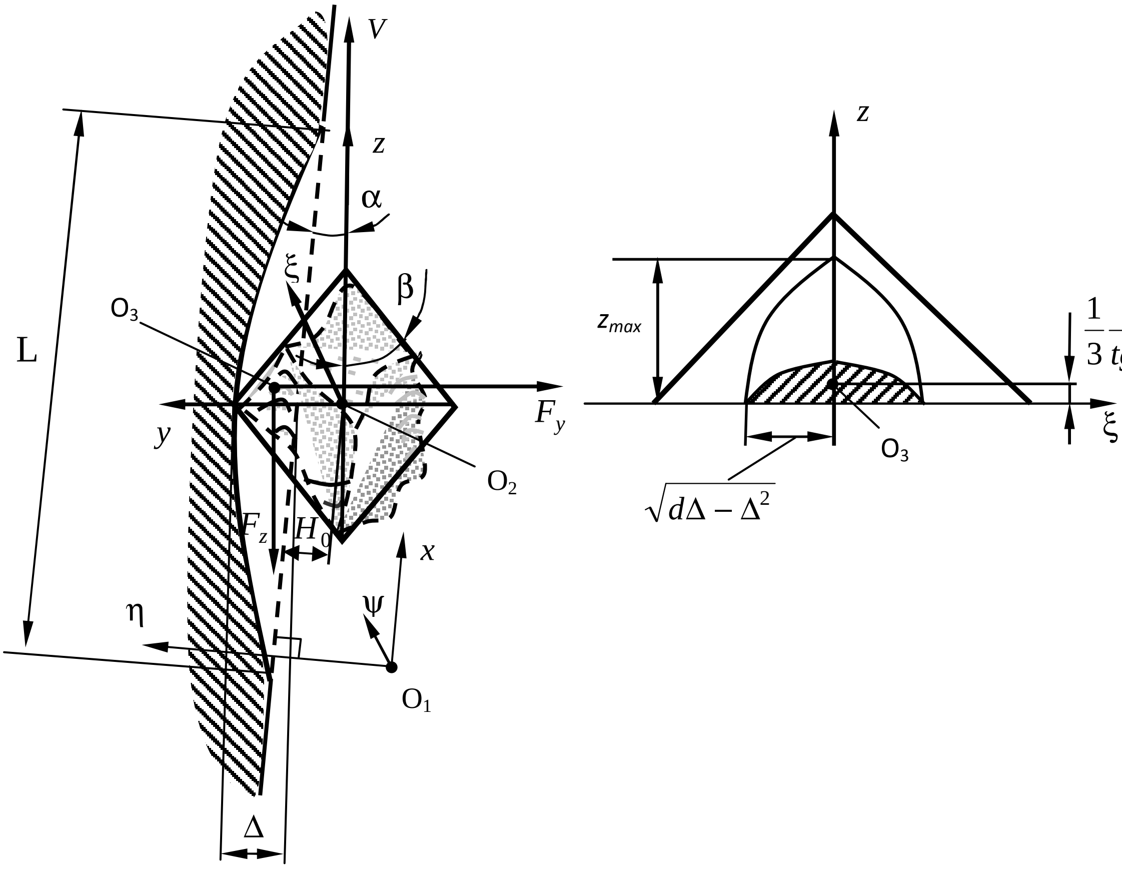



*Fig. 2 Diagram of abrasive particle interaction with surface of work material*

Suppose, that $\xi = 0$ then:

$$z_{\max} = \frac{\Delta}{tg\beta - \sin\alpha}. \qquad (3)$$

When solving the equation (2) at $z$=0, we shall define value $\xi$:

$$\xi = \pm\sqrt{d\Delta - \Delta^2}. \tag{4}$$

Now define the area of a grain projection on plane $y\xi$:

$$S_y = \frac{4}{3}\frac{\Delta\sqrt{d\Delta - \Delta^2}}{tg\beta - \sin\alpha}. \tag{5}$$

In a similar manner we define the projection area of an abrasive grain on plane $z\xi$:

$$S_z = \frac{d^2}{4}\left(\frac{\pi}{180}\arcsin\frac{2\sqrt{d\Delta - \Delta^2}}{d} - \frac{2\sqrt{d\Delta - \Delta^2}}{d}\sqrt{1 - \left(\frac{2\sqrt{d\Delta - \Delta^2}}{d}\right)^2}\right). \tag{6}$$

Taking into account the fact than the whole contact surface is affected by pressure $p$ definable by the formula (1) we find the value of an axial and transverse cutting force (tool pressure):

$$F_y = S_y p = \frac{4}{3}\frac{\sqrt{d\Delta^3 - \Delta^4}}{tg\beta - \sin\alpha}p, \tag{7}$$

$$F_z = S_z p = \frac{d^2}{4}\left(\frac{\pi}{180}\arcsin\frac{2\sqrt{d\Delta - \Delta^2}}{d} - \frac{2\sqrt{d\Delta - \Delta^2}}{d}\sqrt{1 - \left(\frac{2\sqrt{d\Delta - \Delta^2}}{d}\right)^2}\right)p. \tag{8}$$

Define the approximate value of a resulting moment of forces with regard to a centre of mass of an abrasive grain:

$$M = -\left(\frac{d}{2} - \frac{2}{3}\Delta\right)F_z + \frac{1}{3}\frac{\Delta}{tg\beta}F_y. \tag{9}$$

To formulate the equation of an abrasive grain motion we compute the projection of an axial component of the velocity for abscissa and з of immovable co-ordinates *хшз*:

$$\dot{x} = V\cos\alpha, \tag{10}$$

$$\eta = V\sin\alpha, \tag{11}$$

Where $V$ – rate of an abrasive particle

Let us show the expression (1) in the following form:

$$p = \dot{x}^2 k + w, \tag{12}$$

where: $k = \frac{\rho_0}{b(\nu-2)}\left[\frac{\nu-2}{\nu}\left(a^{\frac{\nu}{2}}-1\right)+b(\nu-2)a^{\frac{\nu}{2}}-\left(a^{\frac{\nu}{2}}-1\right)\right]\sin^2\beta,$

$$w = \left(a^{\frac{\nu}{2}}-1\right)\left(p_0+\frac{\tau_0}{\nu(1+\mu)}\right).$$

We put down the equation of motion and a mass centre rotation of an abrasive grain with regard to immovable co-ordinates $x\psi\eta$:

$$\begin{cases} m\ddot{x} = -F_z\cos\alpha + F_y\sin\alpha, \\ m\ddot{\eta} = -F_z\sin\alpha - F_y\cos\alpha, \\ I\ddot{\varphi} = M, \end{cases} \tag{13}$$

where $m$ – mass of an abrasive grain;

$I$ – equatorial inertia moment;

$\varphi = \alpha_0 - \alpha$ - obliquity change of a symmetry axis of a grain to a barrier;

$\alpha_0$ - initial angle between a contact surface and symmetry axis of a grain penetrating.

It is seen from Fig. 2, that:

$$\Delta = \eta - H_0, \tag{14}$$

$$\alpha = \alpha_0 - \varphi. \tag{15}$$

Substituting (14), (15) in (13) we obtain:

$$\begin{cases} m\ddot{x} = \left[\dot{x}^2k+w\right]\left[-\frac{d^2}{4}\gamma\cos(\alpha_0-\varphi)+\frac{4}{3}\lambda\sin(\alpha_0-\varphi),\right], \\ m\ddot{\eta} = -\left[\dot{x}^2k+w\right]\left[\frac{d^2}{4}\gamma\sin(\alpha_0-\varphi)+\frac{4}{3}\lambda\cos(\alpha_0-\varphi),\right], \\ I\ddot{\varphi} = \left[\dot{x}^2k+w\right]\left[-\frac{d^2}{4}\gamma\left(\frac{d}{2}-\frac{2}{3}(\eta-H_0)\right)+\frac{4\Delta}{9tg\beta}\lambda.\right], \end{cases} \tag{16}$$

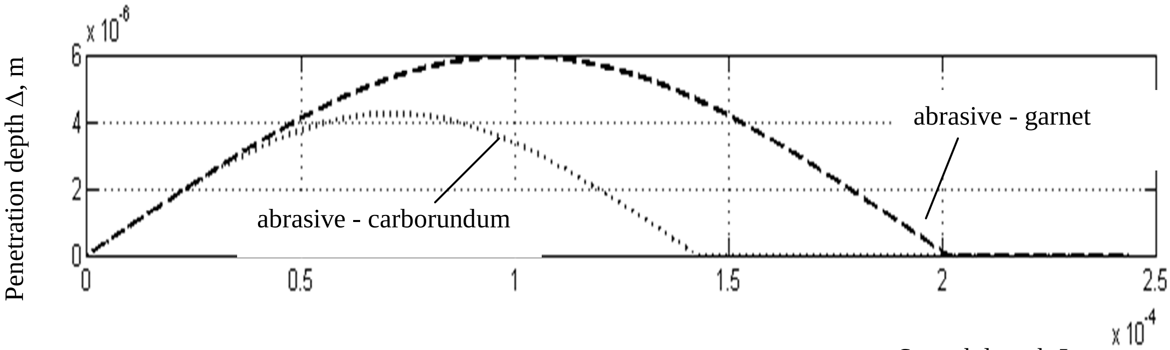
where $\lambda$, $\gamma$ - factors; $\lambda = \frac{\sqrt{d(\eta-H_0)^3-(\eta-H_0)^4}}{tg\beta-\sin(\alpha_0-\varphi)}$;

$$\gamma = \left( \frac{\pi}{180} \arcsin \frac{2\sqrt{d(\eta - H_0) - (\eta - H_0)^2}}{d} - \frac{2\sqrt{d(\eta - H_0) - (\eta - H_0)^2}}{d} \sqrt{1 - \left( \frac{2\sqrt{d(\eta - H_0) - (\eta - H_0)^2}}{d} \right)^2} \right).$$

The initial conditions for the equations set (16);

$$\begin{aligned} \dot{x}(0) &= V_0 \cos\alpha_0, & x(0) &= 0, \\ \dot{\eta}(0) &= V_0 \sin\alpha_0, & \dot{\eta}(0) &= H_0, \\ \dot{\varphi}(0) &= 0. & \varphi(0) &= \alpha_0. \end{aligned} \tag{17}$$

The combined equations (16) are solved through a numerical method. In Fig. 3 there is a result graphically shown.




## 3. Conclusion

The computations carried out show that at the application of garnet - a harder material as an abrasive, the productivity of the process increases in comparison with carborundum by 100-200 % that is proved by comparative tests.

The predictable productivity on rated dependences of a process is good correlated with practical recommendations of manufacturers of hydrocutting equipment for rectilinear cutting with maximum productivity.

### References

1. Stepanov Y., Barsukov G.: Parts surface quality formation at cutting with supersonic liquid stream. „Machine Tool” 2003, Vol. 10, pp. 15 - 17.
2. Stepanov Y., Barsukov G., Mikheyev A.: Barrier destruction with supersonic stream of free abrasive particles. Moskow: Spectrum, 2010.
3. Stepanov Y., Barsukov G., Alexandrov A.: Computational simulation of spherical indenter deflected mode at supersonic impact interaction with a barrier. „Fundamental and applied problems of engineering and technology” 2010, Vol. 2/3. pp. 3-8.