SELECTED ENGINEERING PROBLEMS
NUMBER7
INSTITUTE OF ENGINEERING PROCESSES AUTOMATION
AND INTEGRATED MANUFACTURING SYSTEMS

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ROBOT SX-300 EQUATION, WITH USING NEW GAUSS AND PARAMETRIC METHOD

Abstract: Process of calculation robot Adept SX-300 equation can start from calculation matrix T6 result. My new version of Gauss method take possibility calculate vector of corners, for less complicated equations. For very complicate T6 results, is need additional using parametric method, which can take control values, that are unchanged at the moment of calculation, such as parameters of solution, like constant values. Quality of this solution is very high, and error of calculation is smaller to diameter of electron in hydrogen atom.

1. Introduction

For take robot Adept SX-300 arm function, using T6 method, is need calculate result of translation. Generally, matrix of vectors, on right matrix column, define grapple location. Consist usually, in result, in first line, mostly cosine function, and in second line sine function.

Its correspond to projection on horizontal axis in first line, and vertical axis in second:

$$\begin{aligned} &l1*\cos\theta_1 + l2*\cos\theta_2 + l3*\cos\theta_3 + ... = L \\ &l1*\sin\theta_1 + l2*\sin\theta_2 + l3*\sin\theta_3 + ... = H \end{aligned} \tag{1}$$

Create Gauss matrix:

$$\left[\begin{array}{cccc|c} l1*\cos\theta_1 & l2*\cos\theta_2 & l3*\cos\theta_3 & .... & L \\ l1*\sin\theta_1 & l2*\sin\theta_2 & l3*\sin\theta_3 & .... & H \end{array}\right]. \tag{2}$$

After raise to the second power, both lines, and adding by sides, we obtain result:

$$\begin{aligned} &l1^2*(\cos^2\theta_1 + \sin^2\theta_1) + l2^2*(...) + l3^2*(...) + ... + 2*l1*l2*\cos\theta_1*\cos\theta_2 + ... \\ &+2*l1*l2*\sin\theta_1*\sin\theta_2 + ... = L^2 + H^2 \end{aligned} \tag{3}$$

After moving constant, on right side:

$$2*l1*l2*(\cos\theta_1*\cos\theta_2+\sin\theta_1*\sin\theta_2)+....=L^2+H^2-l1^2-l2^2-l3^2-... \quad (4)$$

After substitution formula: cos (α - β) = cosα*cosβ + sinα*sinβ, we obtain result:

$$2*l1*l2*\cos(\theta_1-\theta_2)+....=L^2+H^2-l1^2-l2^2-l3^2-... \quad (5)$$

For only two corners:

$$\cos(\theta_1-\theta_2)=\frac{L^2+H^2-l1^2-l2^2}{2*l1*l2}. \quad (6)$$

For sample, input matrix:

$$\begin{bmatrix} l1*\cos\theta_1 & l2*\cos\theta_2 & l3|L \\ l1*\sin\theta_1 & l2*\sin\theta_2 & -l4|H \end{bmatrix}. \quad (7)$$

This formula will have form:

$$\cos(\theta_1-\theta_2)=\frac{(L-l3)^2+(H+l4)^2-l1^2-l2^2}{2*l1*l2}. \quad (8)$$

2. Introduction, and apply parametric method

For the same sample of equation, we move constants on right side:

$$\begin{bmatrix} l1*\cos\theta_1 & l2*\cos\theta_2 & |L-l3 \\ l1*\sin\theta_1 & l2*\sin\theta_2 & |H+l4 \end{bmatrix}. \quad (9)$$

For right side we can make new value b1 and b2:

b1= L-l3,

b2= H+l4.

For this values result will equal:

$$\cos(\theta_1-\theta_2)=\frac{b1^2+b2^2-l1^2-l2^2}{2*l1*l2}. \quad (10)$$

End now to this values we can take all corners witch not change in time of calculation. For more complicated matrix T6:

$$\begin{bmatrix} C1*C234 & -C1*S234*S5-S1*C5 & -C1*S234*C5+S1*S5 & C1*(S234*C5*l4+C234*l3+C23*l2+C2*l1)-S1*S5*l4+C1*l0 \\ S1*C234 & -S1*S234*S5+C1*C5 & -S1*S234*C5-C1*S5 & S1*(S234*C5*l4+C234*l3+C23*l2+C2*l1)+C1*S5*l4+S1*l0 \\ S234 & C234*S5 & C234*C5 & -C234*C5*l4+S234*l3+S23*l2+S2*l1+h0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

We can build Gauss Matrix:

$$\left[\begin{array}{cccccc|c} -S1*S6*l4 & C1*S234*C6*l4 & C1*C234*l3 & C1*C23*l2 & C1*C2*l1 & C1*l0 & L \\ C1*S6*l4 & S1*S234*C6*l4 & S1*C234*l3 & S1*C23*l2 & S1*C2*l1 & S1*l0 & Y \\ 0 & -C234*C6*l4 & S234*l3 & S23*l2 & S2*l1 & h0 & H \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]. \quad (12)$$

We can divide first line by C1, and second by S1:

$$\left[\begin{array}{cccccc|c} -S1*S6*l4/C1 & S234*C6*l4 & C234*l3 & C23*l2 & C2*l1 & l0 & L/C1 \\ C1*S6*l4/S1 & S234*C6*l4 & C234*l3 & C23*l2 & C2*l1 & l0 & Y/S1 \\ 0 & -C234*C6*l4 & S234*l3 & S23*l2 & S2*l1 & h0 & H \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]. \quad (13)$$

In next step we can move constants, and parameters on right side equation:

$$\left[\begin{array}{cc|c} C23*l2 & C2*l1 & L/C1+S1*S6*l4/C1-C234*13-l0 \\ C23*l2 & C2*l1 & Y/S1-C1*S6*l4/S1-C234*13-l0 \\ S23*l2 & S2*l1 & H-C234*C6*l4-S234*13-h0 \\ 0 & 1 & 1 \end{array}\right]. \quad (14)$$

We can take, like before, new values b1 and b2:

$$b1 = L/C1 + S1*S6*l4/C1 - C234*l3 - l0, \quad (15)$$

$$b2 = H - C234*C6*l4 - S234*l3 - h0.$$

We can use solution (10) from my version of Gauss method:

$$\cos(\theta_1-\theta_2) = \frac{b1^2+b2^2-l1^2-l2^2}{2*l1*l2}. \quad (16)$$

As solution we have:

$$\Theta2-\Theta23 = \Theta2 - \Theta2 + \Theta3 = \Theta3, \quad (17)$$

$$\theta_3 = a\cos\left(\frac{(L/C1+S1*S6*l4/C1-C234*l3-l0)^2+(H-C234*C6*l4-S234*l3-h0)^2-l1^2-l2^2}{2*l1*l2}\right).$$

Calculation Θ2.

The same we can make with equation (14), for calculation Θ2:

$$C2 = \frac{b1*a1 \; - \; b2*a2}{a1^2 + a2^2}. \tag{18}$$

For a1= l1+l2*C3 and a2=l2*S3:

$$C2 = \frac{b1*(l1+l2*C3) \; - \; b2*S3*l2}{(l1+l2*C3)^2 + (S3*l2)^2}. \tag{19}$$

When we using b1, and b2 full values: (20)

$$\Theta 2 = a\cos\left(\frac{(L/C1+S1*S6*l4/S1-C234*l3-l0)*(l1+l2*C3)-(H-C234*C6*l4-S234*l3-h0)*S3*l2}{(l1+l2*C3)^2+(S3*l2)^2}\right).$$

Calculation Θ1.

For calculation Θ1 we can return to first Gauss matrix (12),

$$\left[\begin{array}{cccccc|c} -S1*S6*l4 & C1*S234*C6*l4 & C1*C234*l3 & C1*C23*l2 & C1*C2*l1 & C1*l0 & L \\ C1*S6*l4 & S1*S234*C6*l4 & S1*C234*l3 & S1*C23*l2 & S1*C2*l1 & S1*l0 & Y \\ 0 & -C234*C6*l4 & S234*l3 & S23*l2 & S2*l1 & h0 & H \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right], \tag{21}$$

and to first two lines, we can take new value a3=S234*C6*l4+C234*l3+C23*l2+C2*l1+l0

$$\left[\begin{array}{cc|c} -S1*S6*l4 & C1*a3 & L \\ C1*S6*l4 & S1*a3 & Y \end{array}\right]. \tag{22}$$

After calculation by my version Gauss method:

$$S6^2*l4^2 + a3^2 - 2*S1*C1*S6*l4*a3 + 2*S1*C1*S6*l4*a3 = L^2+Y^2,$$

$$a3 = \sqrt{L^2 + Y^2 - S6^2 * l4^2}. \tag{23}$$

And after simply calculation S1 value from (22):

$$S1 = \frac{Y*a3 - L*S6*l4}{S6^2*l4^2 + a3^2}. \tag{24}$$

We can take a3 value to equation:

$$\Theta 1 = a\sin(\frac{Y * \sqrt{L^2 + Y^2 - S6^2 * l4^2} - L * S6 * l4}{L^2 + Y^2}). \quad (25)$$

In result we have calculated corners Θ1, Θ2, Θ3.

Removing discontinuity.

For removing discontinuity, we can return to value b1 (15).

b1= L/C1 + S1*S6*l4/C1 – C234*l3 – l0.

Value b1, for corner Θ1= ±90° is going to infinity. For solve this problem, we can see on value a3 in first line of matrix, in calculation a3 (22):

-S1*S6*l4 + C1*a3 = L. (26)

We can calculate a3:

$$a3 = L / C1 + S1 * S6 * l4 / C1. \quad (27)$$

This two discontinuity elements, are equal to a3, and on his place we can take a3 value. In result b1 will change to:

$$b1 = \sqrt{L^2 + Y^2 - S6^2 * l4^2} - C234 * l3 - l0. \quad (28)$$

Now we can change this value in final result, in corners Θ2, Θ3 calculation.

$$\Theta 2 = a\cos(\frac{(\sqrt{L^2 + Y^2 - S6^2 * l4^2} - C234 * l3 - l0) * (l1 + l2 * C3) - (H - C234 * C6 * l4 - S234 * l3 - h0) * S3 * l2}{l1^2 + l2^2 + 2 * l1 * l2 * C3}), \quad (29)$$

$$\theta_3 = a\cos(\frac{(\sqrt{L^2 + Y^2 - S6^2 * l4^2} - C234 * l3 - l0)^2 + (H - C234 * C6 * l4 - S234 * l3 - h0)^2 - l1^2 - l2^2}{2 * l1 * l2}). \quad (30)$$

Now we have calculated corners Θ1, Θ2, Θ3, without discontinuity points.

3. Conclusion

As result of calculation we have corners Θ1, Θ2, Θ3. Else corners are parameters of this equation, and can`t change in time of calculation. This result take possibility using corner Θ6 of grapple rotation. Corner Θ234 in calculation, is equal to corner between l3, and horizontal line. For robots with cornering part of arm l2, like on fig. 1 is need additional calculation of quadratic equation for Θ3. In this article for shorten calculation, l5=0mm.

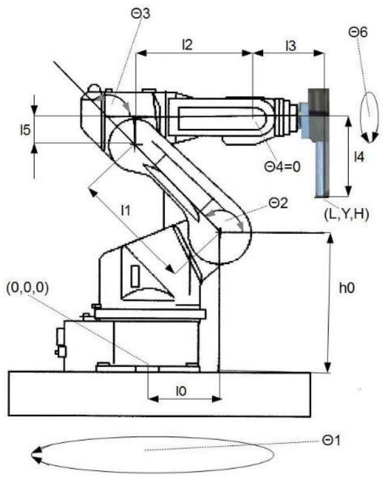

Figure 1. Producer outline scheme of robot Adept SX-300, with angles, and parts of robot arms.

Quality of this calculation is very high, and error of calculation is smaller to diameter of electron in hydrogen atom. So high quality making possibly using this equation, not only for apply for industrial robots, like Adept SX-300, but also to hi quality robots, working in very small raster, for example for microelectronic. Quality of this method, is very important for robotic producers, and for programming, and robotic scientist, designee special development for pharmacy, etc.

Annex: Specification of math symbols, and parameters of robot:

h0 – install height of robot arm over base,
l0 - horizontal displacement, to arm section l1, between Axis1, and Axis2,
l1 - length first arm section between points Axis2 and Axis3,
l2 - length second arm section of robot between points Axis3 and Axis5,
l3 - length third arm section between points Axis5 and Axis6,
l4 - length of grapple in vertical.
Θ1 – angle rotation around Axis1,
Θ2 – angle rotation around Axis2,
Θ3 – angle rotation around Axis3,
Θ4 – angle rotation around Axis5,
Θ6 – angle rotation around Axis6.
L – X coordinate point under grapple,
Y - Y coordinate point under grapple,
H - Z coordinate point under grapple.

References

1. Szkodny T.: Essentials of robotics. Gliwice: Silesian University of Technology publishing house, 2011.